Contemporary accessibility and space syntax research faces an almost symmetrical problem when it comes to analysing and predicting urban pedestrian movement. Both are interested in urban pedestrian movement since it is a crucial issue for the functionality and sustainability of cities, their walkability and liveability (see Jacobs 1961; Williams et al. 2000; Talen 2003; Jenks & Dempsey 2005; EEA 2006), and since globalisation and advancements in information technologies not only ‘eliminate space’ (Miller 2007), in fact it simultaneously makes local places and the design of urban pedestrian space increasingly important (see Castells 1996; Mitchell 1997; McCollough 2004; Florida 2005). Indeed, both research fields express needs for more people-based knowledge that could benefit from one another. In this paper we explore a possibility to combine the space syntax description of the cognitive environment, i.e. topological axial line distance, with conventional description of attraction into an accessibility analysis model that we have called ‘place syntax’ (Ståhle et al. 2005).

Accessibility is broadly (and often vaguely) defined as a measure of an aspect of freedom of action of individuals. It is a very broad concept, and it is related to the concepts of nearness, ease of spatial interaction, and potential of opportunities for interaction (Weibull 1976; 1980). In this section we will focus on attraction-accessibility measures (Miller 2007), but we will show that integration measures, as used in space syntax, cannot technically speaking be viewed as attraction-accessibility measures.

An attraction-accessibility measure is a scalar indicator of accessibility to a specified type of spatial-interaction opportunity $\omega \in \Omega$. Following Weibull (1980) we define an opportunity as represented by a pair $(d_i, w_j)$ where $d_i$ will be called distance, while $w_j$ will be interpreted as a measure of attraction. $\Omega$ is called the universe of opportunities. Here, we will take the point of view of an actor at a given point in geographical space, and we will denote this point in space $i$. An opportunity configuration is a finite unordered list of opportunities, for instance.

Now, we can define a real-valued function $f$ on the set of opportunity configurations and define an accessibility measure $A_i = f(x)$, where $x = (\omega_1, \omega_2, \ldots, \omega_n)$. In practice, one can allow any real-valued function $f$ here (see Weibull 1980), but it is common to impose some restrictions. For instance, with some further assumptions, the accessibility measure can be shown to be additive

$$A_i = T\left(\sum_j g(\omega_j)\right)$$

where $g$ is a real-valued function, and $T$ is a real-valued non-decreasing function. A common representation is given by
$$A_i = \sum_j w_j h(d_{ij})$$

(2)

where it is common to have a simple representation $h(x) = 1/x$.

Hence, to calculate any (attraction-) accessibility measure we need, by definition, to find a finite list of opportunities. Note that, by the axiomatic approach outlined above, there is little guidance as to how to choose attractors and distances (that is opportunities). Instead, the definition of attractors will often (and should) depend on the application at hand, and there are many possible choices of how to measure distance. While Euclidean distances may be viewed as a first approximation (Kwan et al. 2003), other metrics are often used in different contexts. For instance, in the transportation demand literature, it is useful to use a (primal) graph representation, with number of workplaces or shops in the nodes, while the arcs of the graph represent travel time or travel distances, or a combination of both known as generalized cost.

On the other hand, the opportunity configuration may very well be found by a dual representation of such a graph. Although opportunities are embedded in geographical space, distances are then taken to be topological, rather than Euclidian distances (Batty 2004b). In the space-syntax literature, it is common to set attractors all equal to one, and let distances be defined by axial lines. For instance, in the primal representation of a building, corridors may be arcs linking rooms (nodes) together. Here, the distance of a corridor will not be measured by its Euclidian distance, but defined in terms of how one navigates. Hence, an accessibility measure in space syntax is topological rather than an attraction-accessibility as defined above.

In this sense, space syntax deals with ‘spaces’, i.e. featureless spaces in that their attractions are set to unit. On the other hand, it is natural that geographical accessibility deals with ‘places’ that are not featureless, but differ in terms of their content, embodied in their attraction. In fact, Jiang et al. (1999) distinguish between ‘geographic’ and ‘geometric’ accessibility.1

There seems to be some confusion in the literature whether integration analysis, as used in space syntax, can be interpreted as an attraction-accessibility measure. To investigate this, we follow Jiang et al. (1999) and define a structural step distance $S_{ij}$ as the number of steps between two nodes $i$ and $j$ in a graph (which would be a dual graph in the context of space syntax). Using the structural distance we can define an accessibility measure as

$$A_i = \sum_{j \neq i} \frac{1}{S_{ij}}$$

(3)

This is called geometrical accessibility in Jiang et al. (1999).

It is in fact possible to derive this space syntax accessibility measure in eq. (3) from behavioral principles of stochastic effort minimization. Assume that the effort to get to node $j$ from node $i$ can be represented by a random variable $X_{ij}$ with the expected value $S_{ij}$, the number of steps according to the space syntax metric. The randomness of the effort could represent that different actors moving around in the network have different perceptions of the effort (the number of steps) to get from one node to another. Perhaps they have a more or less precise perception of the streets in the network and how they connect to each other. Each actor now tries to find the node in the network that is easiest to get to. An actor located in node $i$ then tries to find the node $j \neq i$ that minimizes his/her effort. This effort will also be a random variable, say $X_j$ that can formally be expressed as $X_j = \min_{i \neq j} X_{ij}$. Assuming further that all $X_{ij}$ are independently and exponentially distributed with expected values $S_{ij}$ respectively, then the accessibility measure $A_i$ according to eq. (3) can be expressed as.
In other words the place syntax accessibility measure (3) is the inverted value of the expected effort to get from a node \( i \) to that node in the network that is associated with the least effort.

In the space-syntax literature, the structural step distance in the dual graph plays a crucial role. Moreover, it is common to use an integration measure to represent how accessible a certain place is. To see the relationship with attraction-accessibility measures, we define mean structural depth (see Jiang et al. 1999) as

\[
D_i = \sum_{j} S_{ij} / n - 1
\]

where \( n \) is the number of nodes in the graph. The integration measure is then defined as (4)

\[
I_i = \frac{n - 2}{2(D_i - 1)}
\]

Clearly, eq. (4) cannot be written in the form of eq. (3). Hence, we conclude that the integration measures used in space-syntax, although being measures of accessibility, cannot be interpreted as attraction-accessibility measures. It should be emphasized that this is, by itself, neither a good nor a bad thing. For instance, an attraction-accessibility measure imposes some restrictions (Weibull 1980). In particular, we cannot take into account that being at a certain place will make it easier to go to another place where accessibility may be different. For instance, attraction-accessibility measures do not take ‘trip chaining’ into account.

It should be clear that what matters for a given accessibility measure is the opportunity configuration, that is the definition of distances and attractors. A primal or dual representation of a graph by itself does not change the accessibility measurement, but using different distances and attractors does. For geometric accessibility, the measurement of distances is critical. On the other hand, in the approach proposed in the paper "Place Syntax" (Ståhle et al. 2005), we have retained the attractors in the accessibility measure. That is, we have used topological distance measures, defined by axial steps (structural depth in a graph defined by axial lines), and at the same time included attractors.

We believe that the marriage between accessibility analysis and space syntax that place syntax represents can bring with it certain fruitful theoretical implications. Our major statement is nonetheless that there is a consistent and fruitful theory of urban space geometry based on human cognition in space syntax. More contributions to detailed accessibility analysis in urban areas, by integration of its three anthropocentric geometries should be expected. Even though the axial line map has been proved to work for pedestrian analysis over and over again, as it also did in our previous study (Ståhle et al. 2005), we wish for even more basic research into its phenomenological, psychological and sociological nature.

Footnotes
1 What is called ‘geometric’ here is close to what is \( F(x) = 1 - \exp(-\lambda x) \) known as ‘pre-geographic’ in for instance Miller (2000).
2 For a more precise definition, see eq. (9) in Jiang et al. (1999)
3 A random variable \( X \) is said to be exponentially distributed with parameter > 0, if its distribution function is \( F(x) = 1 - \exp(-\lambda x) \). Then \( EX = 1/\lambda \). Let for the random variables \( X_i \) be independently and exponentially distributed with parameters \( \lambda_i \), respectively. Then \( \Pr(X^* = \min(X_1, X_2, \ldots, X_n) > x) = \prod \exp (-\lambda x) = \exp(-x\lambda) \). Hence \( 1/EX = \Sigma \lambda_i = \Sigma 1/EX \), which proves the assertion.
4 Although Jiang et al. (1999) in eq. (20) seem to suggest that eqs (3) and (4) are proportional.
References
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